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Experiment on single-mode feedback control of oscillatory thermocapillary convection in a half-zone liquid bridge

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ABSTRACT
Feedback control was carried out on nonlinear thermocapillary convections in a half-zone liquid bridge of a high Prandtl number fluid under normal gravity. In the liquid bridge, the convection changed from a two-dimensional steady flow to a three-dimensional oscillatory flow at a critical temperature difference. Feedback control was realized by locally modifying the free surface temperature using local temperature measured at different positions. The present study aims to confirm whether the control method can effectively suppress oscillatory flows with every modal structure. Consequently, the control was theoretically verified to be effective for oscillatory flows with every modal structure in a high Marangoni number range.
Keywords - Feedback control, half-zone liquid bridge, modal structure, oscillatory flow, thermocapillary convection

I. Introduction

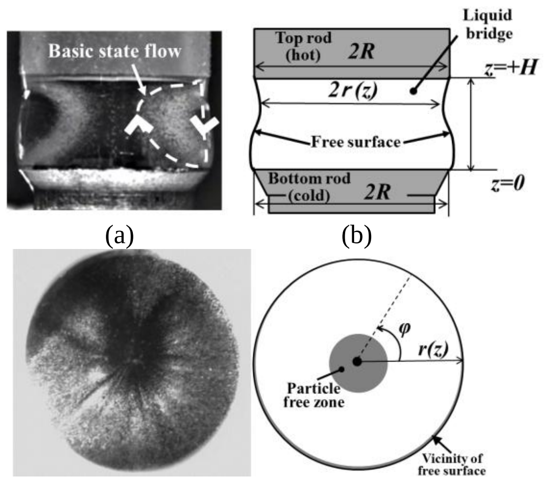
Thermocapillarity has become very important in material processes under microgravity conditions. The float-zone method (FZ) is a material process technique for producing and purifying single crystals of metals and oxides. It is widely known that, using this method, a transition to three-dimensional oscillatory thermocapillary convection takes place at a certain temperature difference between the heated area and cold rods. The oscillatory state of the convection causes detrimental striations in the crystal structure, even under microgravity conditions [1]. This technical problem has inspired a large number of recent studies into the onset of oscillations in thermocapillary convection.

A half-zone model (HZ) has been employed to simplify the thermocapillary convection in FZ. In this model, two co-axial cylindrical rods hold a liquid bridge by its surface tension. By imposing a temperature difference, ΔT, between the rods, thermocapillary flow is induced in the liquid bridge (Fig. 1). The intensity of the induced flow can be described using a Marangoni number, defined as

$$Ma \equiv \frac{\sigma_T \cdot \Delta T \cdot H}{\rho \cdot \nu \cdot \kappa} = \frac{\{\sigma_T \cdot \Delta T/(\rho \cdot \nu)\} \cdot H}{\nu} \cdot \frac{\nu}{\kappa} = \mathrm{Re}_\sigma \cdot \mathrm{Pr} \qquad (1)$$

In the non-dimensional number, σ_T is the absolute value of the temperature coefficient of the surface tension; H, the height of the liquid bridge; ρ, the density; ν, the kinematic viscosity; and κ, the thermal diffusivity. The convection changes from a two-dimensional steady flow to a three-dimensional oscillatory flow at a critical value, ΔT_{cr} or Ma = Ma_{cr} in the case of high Prandtl number (Pr) fluids (Pr > 4) [2, 3]. Overcritical parameter ε is defined as

$$\varepsilon \equiv \frac{Ma - Ma_{cr}}{Ma_{cr}} \qquad (2)$$



(a) (b) (c) (d)

Fig. 1. A half-zone liquid bridge under normal gravity: (a) Side view (picture), (b) side view (outline drawing), (c) top view (picture, z = 1/2H), (d) top view (outline drawing, z = 1/2H), ε < 0.

After the onset of the oscillation, the flow exhibits a modal structure with a certain azimuthal wave number [3, 4]. The dominant mode depends on aspect ratio Γ (Γ= H/R, where R is the radius of the liquid bridge). The mechanism of the onset of the oscillation strongly depends on Pr. Heat transfer plays an important role in a high Pr system [5].

With a better understanding of the phenomena, some results indicated control of oscillatory thermocapillary convection in various geometries. Benz et al. [6] attempted to stabilize the thermocapillary wave instability in an experiment on a plane fluid layer. The temperature signal and phase information detected by thermocouples near the cold end of the layer were fed forward to control a laser that heated the downstream fluid surface along a line. Shiomi et al. [7] and Shiomi and Amberg [8] applied a feedback control based on a simple cancellation scheme for an annular configuration. They modified the surface temperature locally with heating using local surface temperature signals obtained at different azimuthal locations. Using two sensor-actuator pairs, a significant attenuation of oscillation was obtained in a range of ε.

Petrov et al. [9, 10] demonstrated suppression of oscillatory thermocapillary convection through nonlinear feedback control in a half-zone liquid bridge. They modified the surface temperature locally with cooling and heating using local surface temperature signals obtained at different azimuthal locations, and succeeded in completely suppressing the oscillation up to $\varepsilon \approx 0.08$. In our previous research [11], complete damping of the oscillation was achieved using the control method of Shiomi et al. [7, 8] up to $\varepsilon \approx 0.4$ for one of the modal structures.

The present study aims to confirm that our control method can effectively suppress other modal structures. In this paper, we investigate the behavior of surface temperature and flow structure subject to the control.

II. Experiment

2.1 Experimental apparatus

The geometry of the liquid bridge was R = 2.5 mm in radius and aspect ratio Γ = 0.62. The volume of the liquid bridge was virtually equal to the volume of a cylinder ($\pi \cdot R^2 \cdot H$). The radius, R, of the bridge was kept sufficiently small to minimize the buoyancy to thermocapillary force. Silicone oil of 5 cSt (Pr = 68 at 25ºC) was employed as a test fluid (Table 1).

Table 1: Physical properties of silicone oil (KF-96-L, Shinetsu Chemical Co., Ltd)

Silicone oil of 5cSt (25 ºC)	
Density, ρ [kg/m^3]	9.12×10^{2}
Kinematic viscosity, ν [m^2/s]	5.00×10^{-6}
Thermal diffusion coefficient, κ [m^2/s]	7.31×10^{-8}
Surface tension, σ [N/m]	1.97×10^{-2}
Thermal expansion coefficient, β [1/K]	1.09×10^{-3}
Absolute value of temperature coefficient of the surface tension, σ_T [N/m·K]	6.37×10^{-5}
Prandtl number, Pr (= ν/κ) [-]	68.4

The experiment was set up as illustrated in Fig. 2. An axial temperature gradient was imposed by the top and bottom rods. The top rod was heated by a wire heater encircling it, whereas the bottom rod was cooled by the ambient air. The top rod was made of a transparent sapphire glass to observe the flow structure from the top end of the liquid bridge; the bottom rod was made of aluminum. The temperature difference was measured by thermocouples affixed to the top and bottom rods. These experiments were conducted at a temperature of approximately 20 ºC. The flow field was visualized using suspended spherical polystyrene particles with a diameter of 17 μm. The entire bridge was illuminated with a continuous cold light source.

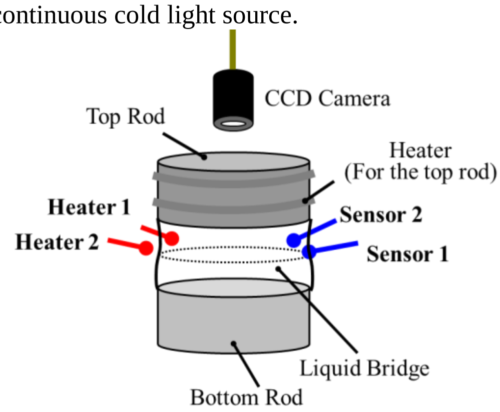


Fig. 2. Experimental setup.

Calibrated cold wires, made of platinum wire with diameter 2.5 μm, were employed as sensors for control. The tips of the sensors were U-shaped with curved bottoms to minimize surface deformation on the liquid bridge, and were approximately 100 μm below the surface. The distance between each supporting prong was 300 μm. In principle, when a constant current is passed through the platinum wire and the resistance is detected, this is proportional to the temperature. Two heaters were employed to heat the surface locally. The heaters were of similar construction to the sensors, but of 10% rhodium-platinum. They were placed approximately 250 μm away from the surface. All sensors and heaters were located at the mid-height of the bridge.

The azimuthal position of sensors and heaters are shown in Fig. 3.

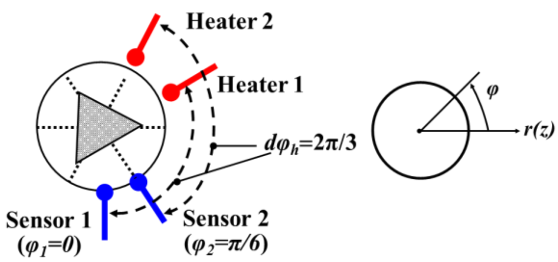


Fig. 3. Sensor and heater arrangements for proportional control of mode-3 oscillation.

2.2 Flow visualization

Following the onset of the oscillation, a polygonal particle-free zone appeared at the center of the r-φ plane. The line of symmetry in the visualized image showed certain polygonal modal flow structures. In this ε, the particle-free zone forms a triangle (Fig. 4). The number of lines of symmetry was three, therefore the oscillation has an azimuthal wavenumber of three (mode-3). Ma_{cr} is approximately 2.2×10^4 at Γ= 0.62. The standing wave type oscillatory flow appear for $0 < \varepsilon \leq 0.9$.

2.3 Control scheme

We applied the control method devised in our previous work [11] to the present work. With Γ = 0.62, the mode-3 oscillation dominated the flow field when control was absent. This means that, at fixed radial and axial positions, surface temperature T (φ, t), T (φ+2π/3, t) will be in phase. A simple cancellation scheme is realized if we introduce point heat sources as follows:

$$Q(\varphi_i + d\varphi_h, t) = -G^*\theta(\varphi_i, t) \quad (\theta(\varphi_i, t) < 0)$$
$$= 0 \quad (\theta(\varphi_i, t) \geq 0) \qquad (3)$$

where φ_i is the i-th azimuthal sensor location and $d\varphi_h$ is the distance between sensors and paired heaters. The dimensionless temperature θ (φ, t) is defined as (T (φ, t) - $\overline{T(\varphi)}$)/ ΔT, where $\overline{T(\varphi)}$ is the time average of T (φ, t). Q and G (≥ 0) are the heater power output and proportional control gain. No cooling device can cool locally and rapidly to cancel oscillation; therefore, a heater was employed.

Since applying only one sensor/heater pair could result in a standing wave with nodes at the sensor/heater positions [10], two sensor/heater pairs were located as shown in Fig. 3.The sensors and heaters were apart by at least π/4. This is because the sensors will receive heat, Q, from the heaters directly if they are placed too close to the heaters.

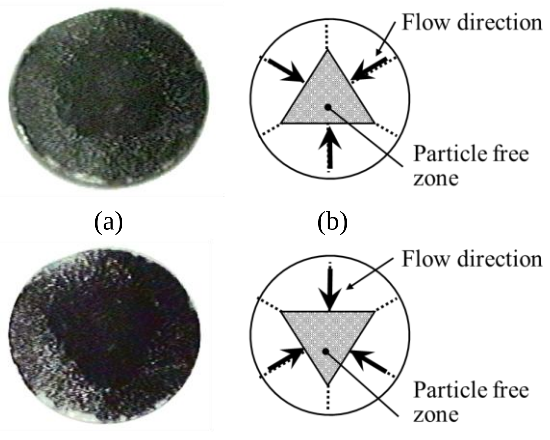


Fig. 4. Top view of flow structure of mode-3 standing wave without control: (a) 0 s (picture), (b) 0 s (outline drawing), (c) 1/2 cycle (0.24 s, picture), (d) 1/2 cycle (outline drawing), Γ= 0.62, ε = 0.5.

III. Results

3.1 Flow without control

In the present aspect ratio, Γ = 0.62, the oscillatory flow of the azimuthal wave number m = 3 (mode-3) appeared after the onset of the oscillatory flow.

Following the onset of the oscillation, the amplitude of the temperature oscillation on the free surface exhibited good agreement with the supercritical Hopf bifurcation in small ε [11]. In this work, the temperature oscillation of mode-3 was also fitted exactly to the supercritical Hopf bifurcation (Fig. 5). Pots of squared amplitudes, θ^2, are apart from the Hopf bifurcation curve in a range of ε > 0.5 for both mode-2 and mode-3. The same relation between θ and ε was found under different experimental conditions [2, 12] in high Pr fluids (Pr > 4).

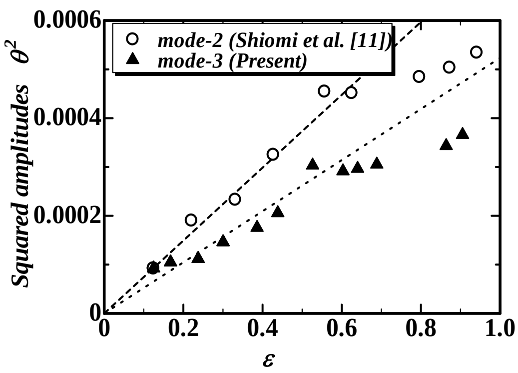


Fig. 5. Squared amplitudes of temperature oscillation, θ^2, for various overcritical parameter ε.

Therefore, the variation of temperature amplitude to ε is a characteristic property for half-zone liquid bridges without relation to modal structures in high Pr fluids. We define the region $0 < \varepsilon < 0.5$ as a weakly nonlinear region and the region $\varepsilon \geq 0.5$ as a nonlinear one.

3.2 Proportional control for single mode

In the weakly nonlinear region (ε < 0.5), the control achieved good performance with complete suppression of the oscillation. Fig. 6 shows the time series of the dimensionless temperature signal θ from one of the sensors.

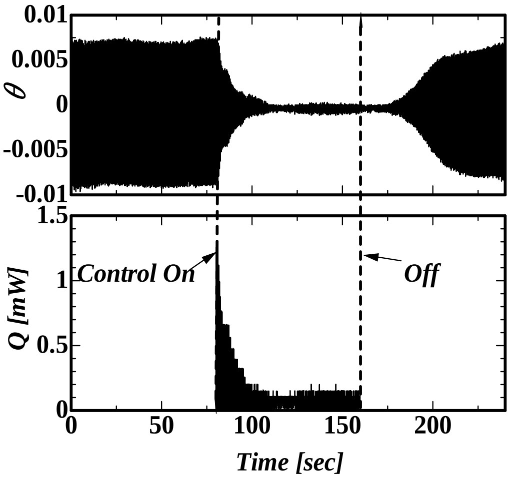


Fig. 6. Time series of temperature amplitude, θ, and heater output power, Q, in weakly nonlinear regime, ε = 0.27.

Turning on the control, temperature amplitude decreased completely within several periods. The heater output initially overshot, then fell to less than 0.5 mW as the control became successful. Similar results were obtained by another sensor.

The global stabilization of the entire flow field can also be observed by flow visualization (Fig. 7). On applying the control, the mode-3 standing wave with the triangle particle-free zone gradually reached a steady axisymmetric state in Figs. 7(a) and (b).

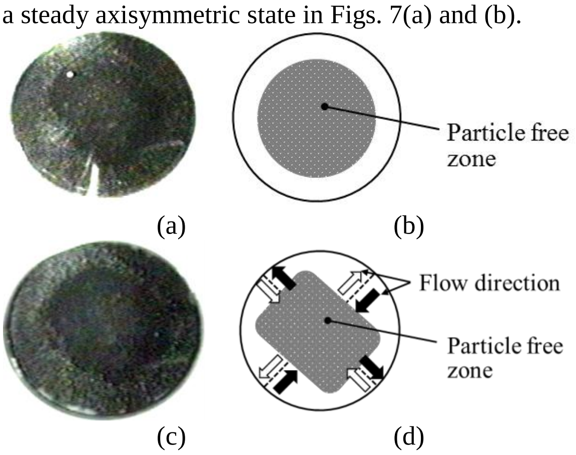


Fig. 7. Top view of flow structure under control: (a) two-dimensional (2D) steady flow (picture), (b) 2D steady flow (outline drawing), (c) mode-2 standing wave (picture), (d) mode-2 standing wave (outline drawing).

In the nonlinear region (ε ≥ 0.5), the performance of the control gradually degraded, even though some attenuation of the oscillation can be observed. This performance degradation resulted from excitation on an oscillation with an expected azimuthal wave number (mode-2).

The new mode-2 oscillation always has a standing structure. It is likely that mode-2 oscillation is excited, since the current control method causes amplification of waves with even wavenumber (Figs. 7(c) and (d)). Shiomi et al. [11] reported that mode-1 oscillation was excited by using the same control manner when the control target was mode-2 oscillation.

Without control, the critical frequency of mode-3 increases as ε increases (Fig. 8). Under control, the critical frequency of mode-3 is virtually the same as that without control. This is because the flow pattern remains unchanged with control. In the present Γ, the flow pattern always indicates a standing wave in ε ≤ 0.9. Once the mode-2 standing wave was excited by the control, the critical frequency decreased 10% of that for mode-3. The frequency of excited mode-2 is approximately 1.5 times that of natural mode-2, frequency in Γ = 0.5.

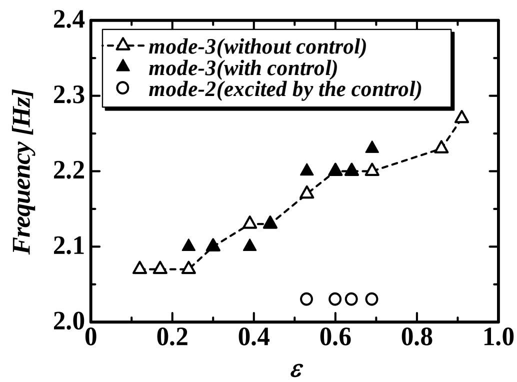


Fig. 8. Critical frequency of temperature oscillation with and without control with mode-3.

To search the optimal proportional gain G, for which maximum attenuation can be obtained, for each ε, a series of experiments with various G was conducted (Fig. 9). To find G_{opt} efficiently, the increments of G were set differently for different ε. θ_{rms} decreased with increasing G and took a minimum value at G_{opt}. For $G > G_{opt}$, θ_{rms} increased with G. The increment of θ_{rms} suggests excitation and amplification of mode-2 standing wave. The G_{opt} increases with ε.

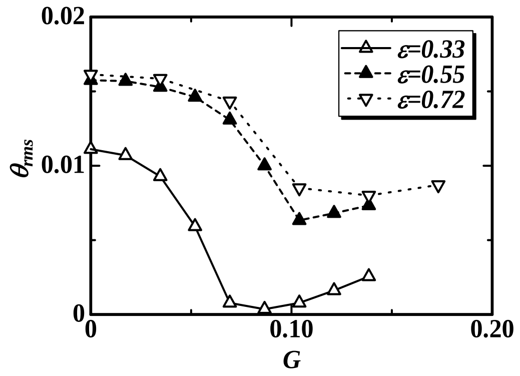


Fig. 9. Non-dimensional temperature amplitude, θ_{rms}, for various proportional gain, G, in different overcritical parameter ε.

The overall performance of the proportional control is shown in Fig. 10. The performance of the control is quantified by the suppression ratio γ_{min}, where γ is the ratio of θ_{rms} with control to θ_{rms} without control. Significant damping of the oscillation was achieved in the range of ε (≤ 0.9). The control showed best performance when ε < 0.41, where oscillation was suppressed down to the level of background noise. The control performance for mode-3 oscillation is virtually the same as that for mode-2, as shown in Figure 10.

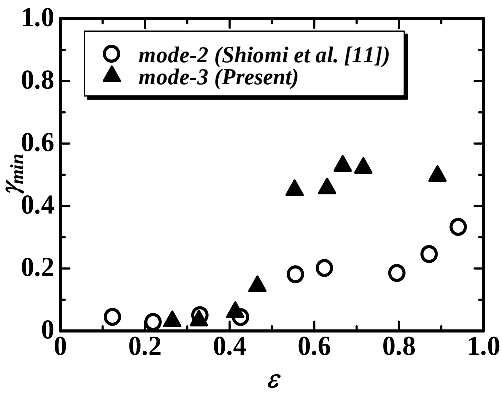


Fig. 10. Suppression ratio of temperature oscillation γ_{min} to various overcritical parameter ε for mode-2 and mode-3.

The control proved to be effective to attenuate the oscillatory convection with the modal structures of both odd and even numbers. In addition, the relation between the temperature amplitude and ε in ε < 0.5 is a characteristic property for half-zone liquid bridges without relation to modal structures in high Pr fluids [2, 12]. Consequently, the control is theoretically effective for oscillatory flows with every modal structure in high Pr fluids. Moreover, excellent performance is achieved in the weakly nonlinear region.

The control method can attenuate only the targeted modal structure depending on the sensor and heater positions. Therefore, in future work, a new control method should be constructed to suppress several modal structures independently of the sensor and heater positions.

IV. Conclusion

A proportional feedback control for single mode was performed for nonlinear thermocapillary convection in a half-zone liquid bridge of high Pr fluid under normal gravity. In the weak nonlinear region (ε < 0.5), suppression of oscillatory flow with mode-3 was completely achieved by the control. In the nonlinear region (0.5 ≤ ε ≤ 0.9), significant attenuation of the oscillation can be obtained. The control method proved to be effective as it suppressed the oscillatory convection with modal structures of both odd and even numbers. Therefore, the control is theoretically effective for oscillatory flows with every modal structure in high Ma range in high Pr fluids.

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